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Performance Comparison of Channel Estimation Technique using Power Delay Profile for MIMO OFDM

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***Abstract*: This paper deals with multiple-input multiple-output orthogonal frequency division multiplexing (MIMO OFDM) system. The performance of a linear minimum mean square error (LMMSE) channel estimator is estimated using power delay profile (PDP) technique. In estimating the power delay profile (PDP), all the pilot symbols must be used at transmit antennas. The insufficient number of samples and null subcarriers causes distortions which leads to inappropriate PDP estimation. So to obtain the accurate PDP the implemented technique holds good. The simulated result using the implemented PDP techniques shows that LMMSE channel estimation performance approaches the wiener filtering.**

Keywords: MIMO, OFDM, power delay profile, channel estimation.

INTRODUCTION:

Multiple input multiple output is the dominant air interface for 4G and 5G broadband wireless communications. MIMO combines multiple input multiple output which multiplies capacity by transmitting different signals over multiple antennas. The idea behind OFDM is to divide the available spectrum into large number of closely spaced subchannels. The combination of MIMO and OFDM technology achieves high spectral efficiency. By using multiple antennas and preceding the data different data streams could be sent over different paths. MIMO at higher speeds would be most manageable using OFDM modulation, because OFDM converts a high speed data channel into a number of parallel, lower speed channels. The advantage of MIMO OFDM is obtaining a spatial diversity which provides considerable gain compared to broadband single antenna. In MIMO OFDM[1] systems channel estimation plays an important role, so it must be known accurately to provide considerable gain at the receiver.

When the receiver knows the channel statistics, then the mean square error is optimized by using LMMSE technique for pilot aided channel estimation. The power delay profile estimation is implemented to obtain the channel statistics[2] at the receiver. OFDM has the advantage of cyclic prefix [10] (CP), because of this advantage maximum likelihood(ML) estimation schemes are used. To obtain the accurate PDP[9] the ML PDP estimator requires complex computation.

By estimating the second-order channel statistics i.e, mean delay and root-mean-square(RMS) delay spread can improve the LMMSE channel estimation performance with approximated PDP. Using pilots the channel delay parameters are estimated with low complex computations .For practical applications like WiMax systems the LMMSE estimator[6] with PDP is appropriate. The correlation mismatch and the error in delay parameters estimation results in performance degradation.

To improve the performance, mismatch must be reduced. The PDP technique implemented in this paper reduces the mismatch. This implemented technique has less computational complexity in estimating the PDP, why because only the pilots from all transmit antenna ports is used. It also reduces the distortion effects caused by insufficient number of estimated channel impulse response(CIR) samples. The simulation results obtained using the implemented technique shows the performance of LMMSE channel estimations[5] approaches the Wiener filtering.

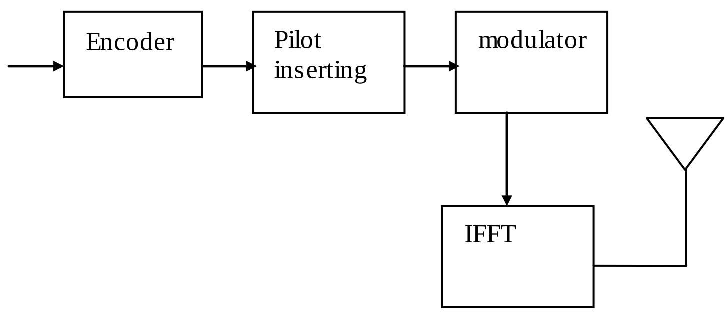

Fig 1: OFDM Transmitter

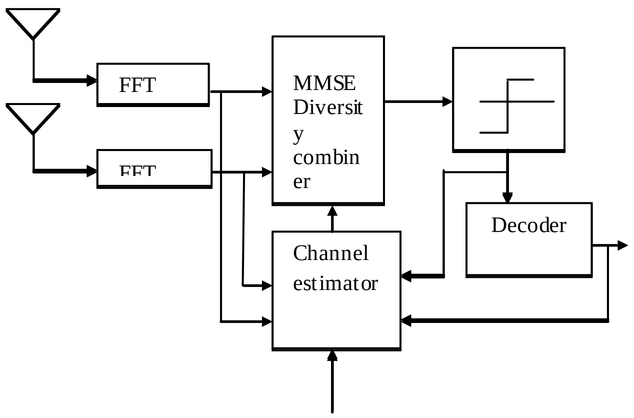

Fig 2: MIMO-OFDM Receiver

SYSTEM MODEL:

The system has P transmit and Q receive antenna MIMO OFDM system is considered with K total subcarriers k_d data subcarriers and K-k_d pilot subcarriers. The analog signal is converted to digital signal and is encoded using the encoder the pilots are inserted to output of encoder. Then the pilot aided OFDM signal is modulated using QAM modulation technique then CP is added to the modulated signal to eliminate the interference. The length of CP is must be longer than channel delay L_{ch}, then only adding CP is beneficial. Adding CP is nothing but the performance of IFFT, after IFFT the obtained output is the OFDM signal. Now it is transmitted to the wireless channel. At the MIMO receiver the reverse operation is performed. CP is removed FFT operation is performed. The corresponding transmit and receive antenna having CIRs has same PDP. At the qth receive antenna the pilot symbol is represented as

$$\boldsymbol{y}_q[n_p] = diag(\boldsymbol{X}_{p)}\boldsymbol{F}_p\boldsymbol{h}_{p,q} + \boldsymbol{n}_q, \ldots\ldots\ldots \quad (1)$$

Where,

$$h_{p,q} = [h_{p,q}[n_p, 0] \ldots h_{p,q}[n_p, L_{ch}], 0 \ldots, 0]^T$$

Which is an L_gX1 CIR vector at pth ,qth transmit and receive antenna.

$$X_p = [C_p[i_1, n_p], C_p[i_2, n_p], \ldots, C_p[i_{kp}, n_p]]^T$$

is a pilot vector at the $n_p th$ OFDM symbol for $i_k \in F_p$ and k=1,2…,K_p.

$diag(\boldsymbol{X}_p)$ is K_pX K_p diagonal matrix whose entries are the K_p elements of the vector X_p.

F_p is a K_pXL_g matrix with the $(i_k$,l)th entry $[F_p]_{i_k}, l = 1/\sqrt{K}$exp{-j2$\pi i_k l/K$} where, $i_k \in F_p$ and l=0,1,….,$L_g - 1$.

$\boldsymbol{n}_q$ = complex additive white Gaussian noise vector (AWGN) which is having zero mean and variance σ_n^2 at each entry at the q^{th} receiver antenna.

IMPLEMENTED METHOD:

Using the regularized least squares (RLS) channel estimation with a fixed length L_g CIR at the (p,q)th antenna port can be estimated as

$$\hat{\boldsymbol{h}}_{R,p,q} = (\boldsymbol{F}_p^H\boldsymbol{F}_p + \in \boldsymbol{I}_{L_g})^{-1}\boldsymbol{F}_p^H\boldsymbol{diag}(\boldsymbol{X}_p)^H\boldsymbol{y}_q[n_p] \triangleq \boldsymbol{W}_{RLS,p}\boldsymbol{y}_q[n_p], \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

Where,

$\in = 0.001$ is a small regularization parameter, and $\boldsymbol{I}_{L_g}$ is an identity matrix with size $L_g X L_g$.

Due to sparsity of pilot tones in frequency domain and the presence of virtual subcarriers makes $\boldsymbol{F}_p^H\boldsymbol{F}_p$ in (2) ill-conditioned. The ensemble average gives the PDP from the estimated CIR obtained from (2). So the ensemble average of $\hat{\boldsymbol{h}}_{R,p,q}\hat{\boldsymbol{h}}_{R,p,q}^{H}$ is

$$E\{\hat{\boldsymbol{h}}_{R,p,q}\hat{\boldsymbol{h}}_{R,p,q}^H\} = WR_{hh}W^H + \sigma_n^2 W_{RLS.p}W_{RLS,p}^H \ldots\ldots \quad (3)$$

Where R_{hh} = E$\{h_{p,q}h_{p,q}^H\}$, and

$$W = (\boldsymbol{F}_p^H\boldsymbol{F}_p + \in \boldsymbol{I}_{L_g})^{-1}\boldsymbol{F}_p^H\boldsymbol{F}_p.$$

The PDP of multipath channel within length L_g is R_{hh} and all the off diagonal elements of R_{hh} are zeros. Hence the covariance matrix can be written as R_{hh} = diag(p_h).

where,

$$p_h = [p_0, p_1, \ldots, p_{L_{ch}}, 0, \ldots, 0]^T \text{ and}$$

$$p_l = E\{|h_{p,q}[n_p, l]|^2\}.$$

Due to the presence of $\boldsymbol{F}_p^H\boldsymbol{F}_p$ in W leads to the distortion in R_{hh}.

So we focus on the method which eliminates the spectral leakage of W, instead of calculating W^{-1}.

The covariance matrix of the estimated CIR is defined as

$$R_{\hat{h}\hat{h}} = WR_{hh}W^H$$

which can be expressed as follows

$$R_{\hat{h}\hat{h}} = \sum_{l=0}^{L_g-1} Wdiag(p_l u_l)W^H, \ldots\ldots\ldots \quad (4)$$

Where

u_l is a unit vector whose lth entry is one, otherwise zeros. Let $p_{\hat{h}}$ and t_l be the L_gX 1 vectors defined as

$p_{\hat{h}} = D_g(R_{\hat{h}\hat{h}})$ and

$t_l = D_g(Wdiag(u_l)W^H)$, respectively,

where $D_g(A)$ is the column vector which contains the diagonal elements of A. Then, the relation in (4) is simplified as

$$p_{\hat{h}} = p_0 t_0 + p_1 t_1 + \cdots + p_{L_g-1}t_{L_g-1} \triangleq Tp_h \quad \ldots\ldots\ldots\ldots\ldots \quad (5)$$

Where T= [$t_0, t_1, \ldots, t_{L_g-1}$] is a distortion matrix caused by W. The distortion matrix satisfies the condition $|[T]_{ii}| > \sum_{j \neq i}|[T]_{ij}|$ for all i,j, which says

that it is a diagonally dominant matrix. The non diagonal elements are leakage powers of u_i for all i. so that the distortion of W can be eliminated as

$p_h = T^{-1} p_{\hat{h}} = E\{g_{p,q}[n_p]\} - \sigma_n^2 \tilde{w}$ …….. (6)

PDP at the (p,q)th antenna port on the n_pth OFDM symbol estimated using the received sample vector $g_{p,q}[n_p]$,

Where $g_{p,q}[n_p] = T^{-1} D_g(\hat{\boldsymbol{h}}_{\boldsymbol{R,p,q}} \hat{\boldsymbol{h}}_{\boldsymbol{R,p,q}}^{\boldsymbol{H}})$ and $\tilde{w} = T^{-1} D_g(W_{RLS.p} W_{RLS,p}^H)$.

PDP ESTIMATION IN PRACTICAL MIMO-OFDM SYSTEMS:

The sample vector received is given as

$g_{p,q}[n_p] = D_g(h_{p,q} h_{p,q}^H) + \tilde{n}_{p,q} + e_{p,q}$,….. (7)

Where

$\tilde{n}_{p,q} = T^{-1} D_g(W_{RLS.p} n_q n_q^H W_{RLS,p}^H)$, and

$e_{p,q} = 2Re\{ T^{-1} D_g(W_{RLS.p} h_{p,q} n_q^H W_{RLS,p}^H)\}$

assume $\tilde{n}_{p,q}$ is an effective noise by AWGN. Then the sample average of received sample vector $g_{p,q}[n_p]$ is given by

$$\langle g_{p,q}[n_p]\rangle_N \triangleq \frac{1}{N}\sum_{n_p}^{|T_p|}\sum_{p=1}^{P}\sum_{q=1}^{Q} g_{p,q}[n_p] \quad \text{…… (8)}$$

$$= \langle D_g(h_{p,q} h_{p,q}^H)\rangle_N + \langle \tilde{n}_{p,q}\rangle_N + \langle e_{p,q}\rangle_N,$$

Where N is the total number of samples for PDP estimation which is given as $N \triangleq |T_p| PQ$.

Where $|T_p|$ = the number of pilot symbols at the k_pth subcarrier in a time slot. To obtain the accurate PDP N must be sufficiently large. Since $\langle D_g(h_{p,q} h_{p,q}^H)\rangle_N \to \boldsymbol{p_h}$, $\langle \tilde{n}_{p,q}\rangle_N \to \sigma_n^2 \tilde{w}$ and $\langle e_{p,q}\rangle_N \to 0$.

It is difficult to obtain such a large number of samples at the receiver in a practical MIMO OFDM system. With an insufficient number of samples, the PDP can be approximated as

$\boldsymbol{p_h} \approx \langle D_g(\boldsymbol{h}_{p,q} \boldsymbol{h}_{p,q}^H)\rangle_N$.

However PDP accuracy with insufficient samples is improved by reducing the effective noise as follows

$$\langle g_{p,q}[n_p]\rangle_N - \sigma_n^2 \tilde{w} = \langle D_g(h_{p,q} h_{p,q}^H)\rangle_N + \boldsymbol{z_N} \quad \text{………. (9)}$$

Where,

$z_N \triangleq \langle e_{p,q}\rangle_N + \langle \tilde{n}_{p,q}\rangle_N - \sigma_n^2 \tilde{w}$ is a residual noise vector, which entry has a zero-mean with N samples the error of PDP estimation can be calculated as

$\tilde{e}_N = (\langle D_g(\boldsymbol{h}_{p,q} \boldsymbol{h}_{p,q}^H)\rangle_N - \boldsymbol{p_h}) + \boldsymbol{z}_N$… (10)

Since, $[\boldsymbol{p_h}]_i \geq 0 \; for \; all \; i$,

$\hat{p}_{init} = \frac{1}{N}\sum_{n_p}^{|T_p|}\sum_{p=1}^{P}\sum_{q=1}^{Q} \boldsymbol{s}_{p,q}[n_p]$, … (11)

Where $\boldsymbol{s}_{p,q}[n_p]$ is the sample vector of implemented PDP estimation with lth entry

$$\boldsymbol{s}_{p,q}^l[n_p] = \begin{cases} g_{p,q}^l[n_p] - \sigma_n^2 \tilde{w}^l & if \; g_{p,q}^l[n_p] > \sigma_n^2 \tilde{w}^l \\ 0 & otherwise \end{cases} \quad \text{…………………. (12)}$$

Where $g_{p,q}^l[n_p] = [g_{p,q}[n_p]]_l$ and $\tilde{w}^l = [\tilde{\boldsymbol{w}}]_{\boldsymbol{l}}$.

The implemented technique estimates the average of residual noise at the zero-taps of $\boldsymbol{p_h}$, to reduce the effect of residual noise $\boldsymbol{z}_N$. The zeo-tap is detected at the lth entry of $\hat{p}_{init}$ as

$$t_z^l = \begin{cases} 1 & if \; \hat{p}_{init}^l < \beta_{th} \\ 0 & otherwise \end{cases} \quad \text{…….. (13)}$$

Where $\beta_{th} = \frac{1}{L_g}\sum_{l=0}^{L_g-1} \hat{p}_{init}^l$ is threshold value for detecting zero-tap. At zero-taps the average of residual noise is estimated as

$\hat{n}_{R,avg} = \frac{1}{N_z}\sum_{l=0}^{L_g-1} \hat{p}_{init}^l \; t_z^l$ …… (14)

Where

$N_z = \sum_{l=0}^{L_g-1} t^l$ is the number of detected zero-taps. The PDP estimate $\hat{\boldsymbol{p}}_h$ at the lth tap after reducing the residual noise, can be expressed as,

$$\hat{p}_h^l = \begin{cases} \hat{p}_{init}^l - \hat{n}_{R,avg} & if \; \hat{p}_{init}^l > \hat{n}_{R,avg} \\ 0 & otherwise \end{cases} \quad \text{……….. (15)}$$

The frequency-domain channel correlation in the LMMSE channel estimator is obtained using the estimated PDP in (15).

PERFORMANCE ANALYSIS:

With imperfect PDP in (15) the LMMSE channel is estimated as

$$\boldsymbol{W}_{f,p} = \boldsymbol{F}_L D_g(\hat{\boldsymbol{p}}_h)\boldsymbol{F}_p^H (\boldsymbol{F}_p D_g(\hat{\boldsymbol{p}}_h)\boldsymbol{F}_p^H + \sigma_n^2 \boldsymbol{I}_{k_p})^{-1} \quad \text{……………. (16)}$$

Where by taking first L_g columns of DFT matrix $\boldsymbol{F}_L$ which is K_d X L_g matrix is obtained. $\hat{\boldsymbol{p}}_h = \boldsymbol{p}_h + \bar{e}_{pdp}$ is expressed as the estimated PDP. The $\bar{e}_{pdp}$ for the lth element is defined as

$$\bar{e}_{pdp}^l = \begin{cases} [\tilde{e}_N]_l - \hat{n}_{R,avg} & if \; [\tilde{e}_N]_l > \hat{n}_{R,avg} \\ 0 & otherwise \end{cases} \quad \text{……………. (17)}$$

From matrix inversion lemma, $\boldsymbol{F}_p D_g(\hat{\boldsymbol{p}}_h)\boldsymbol{F}_p^H + \sigma_n^2 \boldsymbol{I}_{k_p})^{-1}$ is converted as

$$\boldsymbol{F}_p D_g(\hat{\boldsymbol{p}}_h)\boldsymbol{F}_p^H + \sigma_n^2 \boldsymbol{I}_{k_p})^{-1} = A^{-1} - A^{-1}\boldsymbol{F}_p B \boldsymbol{F}_p^H A^{-1} \quad \text{………………. (18)}$$

Where $A \triangleq \boldsymbol{F}_p D_g(\hat{\boldsymbol{p}}_h)\boldsymbol{F}_p^H + \sigma_n^2 \boldsymbol{I}_{k_p})^{-1}$

$B \triangleq D_g(\bar{e}_{pdp})(I_{L_g} + \boldsymbol{F}_p^H A^{-1} F_p D_g(\bar{e}_{pdp}))^{-1}$.

Then, the coefficient matrix for LMMSE channel estimation with $\hat{p}_h$ can be rewritten as

$$W_{f,p} = W_{opt,p} + W_{err,p}, \quad \ldots\ldots\ldots\ldots \quad (19)$$

Where

$$W_{opt,p} \triangleq \boldsymbol{F}_L D_g(p_h)\boldsymbol{F}_p^H(\boldsymbol{F}_p D_g(p_h)\boldsymbol{F}_p^H + \sigma_n^2 \boldsymbol{I}_{k_p})^{-1}$$

is the coefficient matrix for wiener filtering, and

$$W_{err,p} = -\boldsymbol{F}_L D_g(p_h)\boldsymbol{F}_p^H A^{-1}\boldsymbol{F}_p B \boldsymbol{F}_p^H A^{-1} + \boldsymbol{F}_L D_g(\bar{e}_{pdp})\boldsymbol{F}_p^H \boldsymbol{F}_p D_g(\hat{\boldsymbol{p}}_h)\boldsymbol{F}_p^H + \sigma_n^2 \boldsymbol{I})^{-1} \quad \ldots\ldots\ldots\ldots \quad (20)$$

With imperfect PDP the error covariance matrix of LMMSE channel estimation can be obtained as

$$E_p = E\{(F_L h_{p,q} - W_{f,p}\hat{h}_{LS,p,q})(F_L h_{p,q} - W_{f,p}\hat{h}_{LS,p,q})^H\}$$

$$=(F_L - W_{f,p}\boldsymbol{F}_p)\,D_g(p_h)(F_L - W_{f,p}\boldsymbol{F}_p)^H + \sigma_n^2 W_{f,p}\boldsymbol{F}_p\boldsymbol{F}_p^H W_{f,p}^H, \quad \ldots\ldots\ldots\ldots \quad (21)$$

Where $\hat{h}_{LS,p,q} \triangleq diag(X_p)^H \boldsymbol{y}_p[n_p]$

Using the error covariance matrix the frequency – domain MSE of the implemented method is

$$MSE_{f,p} = \frac{Tr(E_p)}{Tr(F_L D_g(p_h)F_L^H}, \quad \ldots\ldots\ldots\ldots \quad (22)$$

RESULTS AND DISCUSSION:

The simulation results obtained in MIMO OFDM system with two transmit and receive antennas. The simulated graph shows the channel estimation by taking different parameters. The simulated graphs show the MSE performance of LMMSE technique using estimated PDP.

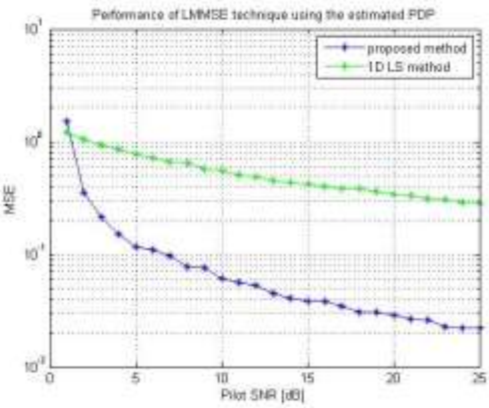

Fig 3: Graph plotted between MSE and SNR using implemented MMSE and 1D LS method.

In figure 3 the graph shows the relation between MSE and SNR. As the MSE is decreasing then the SNR is increasing i.e., as the mean square error is reducing the signal level is increasing which leads to increase in SNR ratio. In figure 4, graph plotted between MSE and channel length. As the channel length is increasing the MSE also increases the interference get increases. In figure 5, graph shows

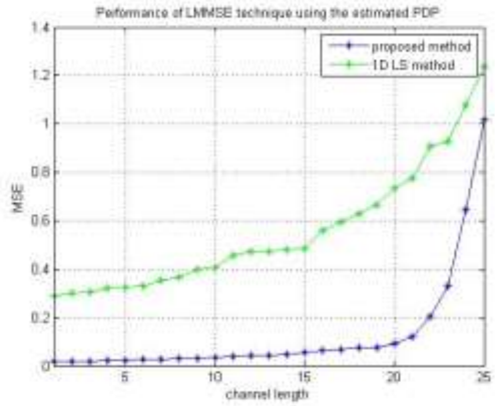

Fig 4: Graph plotted between MSE and channel length using implemented MMSE and 1D LS method.

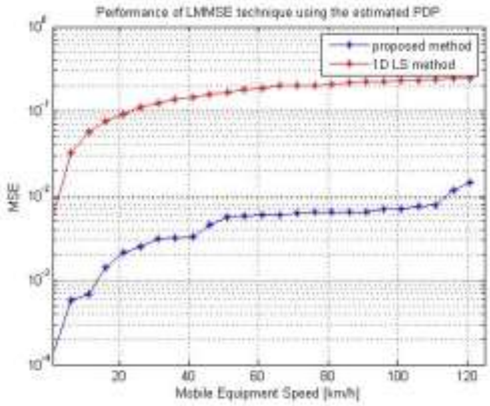

Fig 5: Graph plotted between MSE and mobile equipment speed using implemented MMSE and 1D LS method.

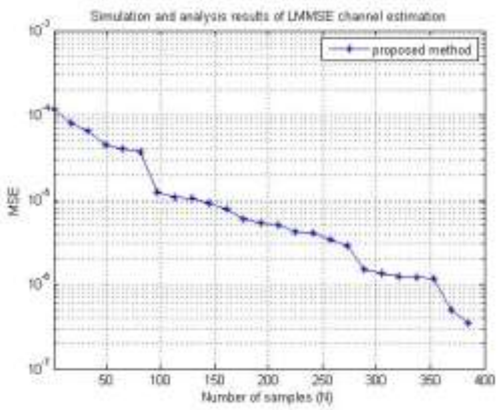

Fig 6: Graph plotted between MSE and number

of samples using implemented MMSE and 1D LS method.

how the MSE varies with the mobile equipment speed. As the mobile is moving fast then it experiences a fast fading in multipath channel which increases the MSE. In figure 6 as the number of samples increases the MSE decreases.

CONCLUSION:

The simulated graphs show the comparison between the implemented method using LMMSE technique and 1D least square technique. The implemented technique using LMMSE gives less mean square error compared to the 1D least square method. Implemented algorithm holds good, as the number of samples increases also observed from the result. The implemented technique effectively reduces the spectral leakage and residual noise. The performance of LMMSE channel estimation approaches that of wiener filtering using the implemented PDP estimate.

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